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Static bending and buckling analysis of functionally porous beam by First order Shear Deformation Theory

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KEYWORDS

Porous beam,
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ABSTRACT

In this paper, static bending and buckling due to different distributions of functional porosity based on Timoshenko's beam theory are investigated. The modulus of elasticity and mass density are considered according to the two patterns of specific porosity distribution in the direction of thickness. The partial differential governing equations are derived from the minimum potential energy principle. The Ritz method is used to calculate the critical buckling loads and transverse bending curvature. The obtained results have been compared with other references and also finite element modeling. A parametric study was performed to investigate the effects of porosity coefficient and slenderness ratio on buckling and flexural properties of porous beams with pinned boundary conditions. Also, the effect of different porosity distributions on structural performance has been investigated to obtain essential insights into the design of this type of beam to achieve the desired buckling strength and flexural behavior. According to the parametric study results, increasing the porosity coefficient and slenderness ratio increase the critical buckling load and the bending strength. This research results can design porous beams like metal foams or recent manufacturing methods like additive manufacturing. Among the applications of the present study, this type of porous material for bone repair scaffolding can be mentioned.

1. Introduction

Structures made of functionally graded porous materials because of their unique benefits provided by the distribution of heterogeneous properties have been considered by researchers. Many studies have been published in this field in the last few decades [1-11].

The present paper considers a functionally graded porous beam with two porosity distributions in the thickness direction. The effects of porosity coefficient and slenderness ratio are studied.

2. Methodology

A beam with length L and thickness h with two types of porosity distributions specified in the direction of thickness is introduced in this section. This beam is illustrated in the Cartesian coordinate system in Figure 1. Both porosity distributions have the same maximum and minimum values of elastic modulus and mass density.

Concerning non-uniform porosity distribution, E Young's modulus, G Shear modulus, and ρ mass density for first type porosity distribution are defined in equations 1 to 3. Also, the porosity distribution of the second type is defined as equations 4 to 6.

$$E(z) = E_1[1 - e_0 \cos(\pi\zeta)] \tag{1}$$

$$G(z) = G_1[1 - e_0 cos(\pi\zeta)] \tag{2}$$

$$\rho(z) = \rho_1[1 - e_m cos(\pi\zeta)] \tag{3}$$

$$E(z) = E_1[1 - e_0 cos(\frac{\pi}{2}\zeta + \frac{\pi}{4})] \tag{4}$$

$$G(z) = G_1[1 - e_0 cos(\frac{\pi}{2}\zeta + \frac{\pi}{4})] \tag{5}$$

$$\rho(z) = \rho_1[1 - e_m cos(\frac{\pi}{2}\zeta + \frac{\pi}{4})] \tag{6}$$

In equations 1 to 6, $e_0 = 1 - \frac{E_0}{E_1} = 1 - \frac{G_0}{G_1}$. The value of E_1 shows the Young modulus at the upper and lower levels of the beam (z = ±h/2) in the first type porosity distribution. In the porosity distribution of the second type, E_1 indicates the upper surface of the beam (z =h/2). Equation 7 shows the relationship between Young's modulus and the shear modulus. In this analysis, Poisson's ratio is considered the same for the whole material.

$$G_i = \frac{E_i}{2(1+\nu)} \quad (i = 0,1) \tag{7}$$

In this study, the first-order shear deformation theory for beams has been used. Due to the inhomogeneity, the effect of shear in the direction of thickness can not be ignored. According to first-order shear deformation beam theory, the displacement field is used as Equation 8 to calculate the transverse shear strain effect of the beam.

$$\begin{cases} u(x,z,t) = u_0(x,t) + z\phi_x(x,t) \\ w(x,z,t) = w_0(x,t) \end{cases} \tag{8}$$

u_0 and w_0 indicate the axial and transverse displacement in the center of the beam, respectively and ϕ_x is the rotation of the cross-sectional area of the beam. Equations 9 and 10 define the linear strain-displacement relations of the beam.

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} + z\frac{\partial \phi_x}{\partial x} \tag{9}$$

$$\gamma_{xz} = \frac{\partial w_0}{\partial x} + \phi_x \tag{10}$$

The stress-strain relations are defined in relations 11 to 13. Q is a function of E (Young's modulus) and υ (Poisson's ratio).

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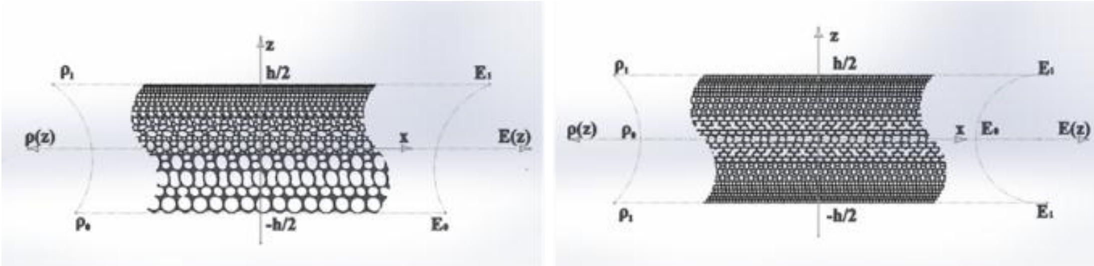

(a) (b)

Figure 1. porosity distribution, a) type 1, b) type 2.

$$\sigma_{xx} = Q_{11}(z) \times \varepsilon_{xx} \quad (11)$$

$$\sigma_{xz} = Q_{55}(z) \times \gamma_{xz} \quad (12)$$

$$Q_{11}(z) = \frac{E(z)}{1-\upsilon^2}; \quad Q_{55}(z) = G(z) = \frac{E(z)}{2(1+\upsilon)} \quad (13)$$

By replacing equations 9 to 13 in the strain energy equation U and then using equation 8, the strain energy of the porous beam is obtained from equation 14.

$$U = \frac{1}{2}\int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})dzdx \quad (14)$$

The beam is loading longitudinally and transversely along the axis z-direction. The work done by these loads is given in Equation 15.

$$V = \int_0^L [\frac{1}{2}N_{x0}(\frac{\partial w_0}{\partial x})^2 + Qw_0]\, dx \quad (15)$$

In the above equation, N_{x0} is the axial force in the direction of the beam, and Q shows the transversely distributed load on the upper plane of the beam (z =h/2). According to Hamilton's principle, the total energy of a porous beam is obtained as Equation 16.

$$\Pi = U - V \quad (16)$$

The unknown functions of Equation 8 and the critical buckling load values are obtained using the Ritz method. In order to obtain the displacement field, the algebraic functions are used in the form of algebraic polynomials in the form of equation 17, which satisfies the boundary conditions of the simply supported beam.

$$\begin{cases} u(x) = \sum_{j=1}^{N} A_j x^j (1-x) \\ w(x) = \sum_{j=1}^{N} B_j x^j (1-x) \\ \varphi(x) = \sum_{j=1}^{N} C_j x^{j-1} \end{cases} \quad (17)$$

N represents the number of simple algebraic polynomials obtained by changing the number of buckling modes, and the coefficients A_j, B_j, and C_j are unknown values. According to the Ritz method, the unknown coefficient can be obtained by minimizing energy as Equation 18.

$$\frac{\partial \Pi}{\partial A_j} = 0, \frac{\partial \Pi}{\partial B_j} = 0, \frac{\partial \Pi}{\partial C_j} = 0 \quad (18)$$

Equation 18 is a system of algebraic equations that, by solving this system, unknown coefficients are obtained. For the non-zero response of this algebraic relations device, the determinants of the coefficients are zero. The answer to this step is equal to the critical buckling load of the porous beam. The dimensionless buckling critical load is obtained by dividing the axial force into the components of the homogeneous beam stiffness matrix according to Equation 19:

$$P_{cr} = \frac{N_{x0}}{\frac{E_1 h}{1-\nu^2}} = \frac{N_{x0}(1-\nu^2)}{E_1 h} \quad (19)$$

3. **Discussion and Results**

Figure 2 shows critical buckling load in a functionally graded porous beam with symmetric and asymmetric porosity distribution. The critical buckling load varies with slenderness ratio and porosity coefficient. The critical buckling load decrease with increasing the porosity coefficient and slenderness ratio. There is a more significant effect with porous coefficient in lower slenderness ratio. Also, the distribution of symmetric porosity has a more buckling resistance than the distribution of asymmetric porosity. Therefore, as expected, the internal porous reduce the effective stiffness of the beams and increase the slenderness ratio reduces the buckling strength of the beams.

Figure 3 shows the porosity effect and slenderness ratio on the maximum deflection of the beam. The dimensionless displacement is obtained by dividing the transverse displacement by beam height. The maximum displacement occurs with the maximum porosity. There is more displacement in the asymmetric porosity pattern and a low effect in the porosity pattern in a lower slenderness ratio. There is more difference between porosity pattern and porosity coefficient in higher slenderness ratio.

A comparison of analytical and numerical results in terms of uniform distribution load is shown in Table 1. The difference between numerical and analytical results is 15.86%.

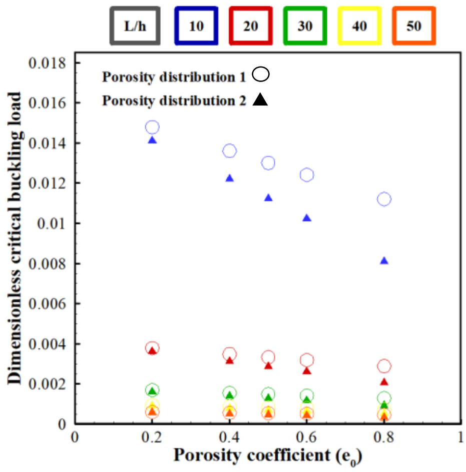

Figure 2. the effects of porosity coefficient and slenderness ratio on critical buckling load of functionally graded porous beams

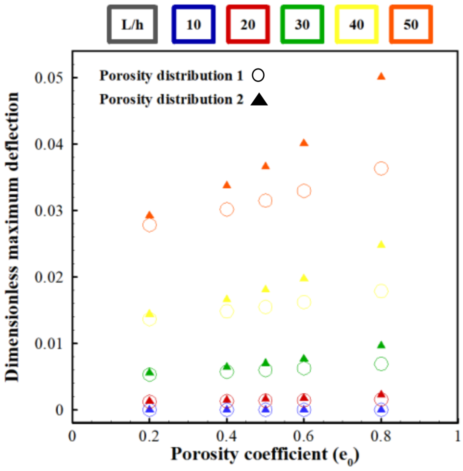

Figure 3. the effects of porosity coefficient and slenderness ratio on maximum deflation of functionally graded porous beams

Table 1. Maximum deflection of beam

Method	Deflection
Analytical method	0.001368
FEM (Abaqus)	0.001151

4. **Conclusions**

This paper investigated the elastic buckling and static bending of functionally graded porous beams with specific porosity distributions. The governing equations were obtained by applying First Order Shear Deformation Theory and Hamilton principle. The critical buckling load and transverse deflection receive by the Ritz method. Numerical results show the effect of porosity coefficient and slenderness ratio on the critical buckling load and maximum deviation. (1) the critical buckling load of functionally graded porous beams reduces by increasing the porosity coefficient and slenderness ratio. (2) The maximum deflection for porous beams increases with increasing porosity coefficient and slenderness ratio. As the porosity decreases, the beam becomes denser, so the beam can withstand more loads. (3) The porosity distribution pattern has a significant effect on the buckling and flexural behavior of the beam. Compared to the asymmetric distribution model, the symmetrical distribution offers better buckling capacity and flexural strength. The porosity distribution plays a more critical role in flexural behavior and buckling strength at a larger slenderness ratio. So, the appropriate porosity distribution can be designed according to the objectives and application of the model. Porosity coefficient, beam slenderness ratio, and porosity distribution affect static bending and critical buckling load in designing items such as bone scaffolds. Compressive strength and flexural bending, and porosity for bone growth are essential parameters in designing such materials.

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